Problem Set 7.3 – Complex numbers in exponential form

1. Evaluate the following in rectangular, polar and exponential forms. Give answers to 3 sig. figs.

(a) $(3+5j)^5$

(b) $(5+j3)(7+j2)$

(c) $(7-j2)(-1+j7)$

(d) $(7-j2)+5\angle 75^{\circ}$

(e) $(2e^{j7})+(7e^{j2})$

(f) $(6e^{j5})(7e^{-j2})$

(g) $(6e^{j5})-(7e^{-j2})$

(h) $(6e^{j})+(5+j10)$

(i) $(5\angle 45^{\circ})(6e^{j\pi})$

2. Find the required real or imaginary part of each complex expression to 3 sig. figs.
Hint for (c): The exponent must be split apart like this: $e^{(-5+12j)t}=e^{-5t}\cdot e^{12jt}=(e^{-5t})\angle(12t)$.
As t increases this traces out an inward spiral in the complex plane.

(a) $\text{Im}\left\{\dfrac{5e^{2jt}}{6+8j}\right\}$

(b) $\text{Re}\left\{\dfrac{e^{10jt}}{-5+12j}\right\}$

(c) $\text{Re}\left\{\dfrac{e^{(-5+12j)t}}{-5+12j}\right\}$

(d) $\text{Im}\left\{\dfrac{e^{(5+12j)t}}{5+12j}\right\}$

3. Find the roots of the complex equation $z^4 = 9.85 + 12.6j$ and show them in the complex plane.
Hint: They can be found with De Moivre's theorem.

4. Make a sketch of $e^{(1+12j)t}$ in the complex plane for $0 \le t \le 2$.

5. By adding and subtracting Euler's formula and its complex conjugate show that

$$\cos\theta = \frac{e^{j\theta}+e^{-j\theta}}{2} \quad \text{and} \quad \sin\theta = \frac{e^{j\theta}-e^{-j\theta}}{2j}$$

Interpret the terms of cos and sin as pairs of points moving in the complex plane as θ increases.

6. In chapter 4, page 152 we made a table of values showing that $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e^1 = e$. In fact an alternative definition of the exponential function is $e^x = \lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n$, where x can be any complex number. Construct an Excel spreadsheet that takes x as input and makes a table similar to the one on page 152. Hint: set up columns like this and let n run from 1 to, say, 100:

n	$\left(1+\frac{x}{n}\right)$	$\left(1+\frac{x}{n}\right)^n$	$\text{Re}\left(1+\frac{x}{n}\right)^n$	$\text{Im}\left(1+\frac{x}{n}\right)^n$

Then plot the last two columns in a scatter plot. The figure shows the plot for two values of x: $x = 1$ and $x = i\pi$. We clearly see the limits.
Tip: Excel uses various character string functions to do complex arithmetic. Use these:
=complex to create a complex number,
=impower to take an integer power of it,
=imreal to take the real part,
=imaginary to take the imaginary part.