Example 4.20: Suppose that we put \$1500 into a bank account which receives interest at a rate of 8% per year and which is compounded continuously. (*Compounded continuously* is just another way of saying *grows exponentially*.) Let y denote the amount of money in the account at any time t. Then y can be expressed in the "rate form" (4.32),

$$y = y_0 \cdot e^{rt},$$

where $y_0 = \$1500$ is called the *principal* and $r = 0.08/\text{yr}$ is the interest rate. The units of y will also be dollars and t must be given in years so that the units of r and t cancel. (a) What amount will be in the account at the end of 15 months? (b) After how many years will there be \$4000 in the account?

Solution:

(a) Letting t = 15 months = 1.25 yrs we find that:

$$y \quad = \quad \$1500 \cdot e^{(0.08/\text{yr})(1.25\,\text{yr})} \quad = \quad \$1657.75 \qquad \leftarrow \text{answer}$$

(b) Letting $y = \$4000$ and solving the equation for t gives

$$\$4000 = \$1500 \cdot e^{(0.08/\text{yr}) \cdot t}$$

$$\ln\left(\frac{\$4000}{\$1500}\right) = \frac{0.08}{\text{yr}} \cdot t$$

$$t = 12.26 \text{ yrs.} \qquad \leftarrow \text{answer}$$

Note: This example nicely illustrates the fact that in the functions $\ln(x)$ and e^x the argument x must be dimensionless (it cannot have units). We saw this in $e^{(0.08/\text{yr})(1.25\,\text{yr})}$ and $\ln\left(\frac{\$4000}{\$1500}\right)$.