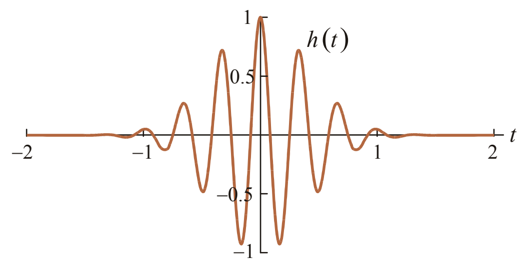
To summarize: given a waveform $h(t)$, we use Eq. (10.82) to find its Fourier transform, $H(f)$, which tells us what frequencies are present in the waveform. On the other hand, given the Fourier transform $H(f)$, we can use (10.81) to carry out the **inverse Fourier transform** and get back the original waveform, $h(t)$.

Example 10.14: Consider the waveform $h(t)=e^{-3t^2}\cos(6\pi t)$ shown in Fig. 10.36. This shape is called a wave packet. The factor $\cos(6\pi t)$ causes the waveform to oscillate at 3 Hz and the factor e^{-3t^2} (called a Gaussian envelope) causes it to exist for only a brief time. Hence $h(t)$ is not periodic. Calculate its Fourier transform. Write an Excel Visual Basic program that uses Simpson's rule to do the integrals and use Excel to plot the graph.

Figure 10.36 This waveform contains a 3Hz oscillation but exists only for a brief time and hence is not periodic.



Solution: We must carry out the integral $H(f)=\int_{-\infty}^{\infty} h(t)e^{j2\pi f t}\,dt$ in (10.82) and plot the resulting $H(f)$. Generally $H(f)$ is a complex function with a real part and an imaginary part to plot but because $h(t)$ is an even function the imaginary part of $H(f)$ is zero. Here is the proof:

$$
\begin{aligned}
H(f) &= \int_{-\infty}^{\infty} h(t)e^{j2\pi f t}\,dt \\
&= \int_{-\infty}^{\infty} e^{-3t^2}\cos(6\pi t)\cdot\big(\cos(2\pi f t)+j\sin(2\pi f t)\big)\,dt \\
&= \underbrace{\int_{-\infty}^{\infty} e^{-3t^2}\cos(6\pi t)\cos(2\pi f t)\,dt}_{\text{even function}} \;+\; j\int_{-\infty}^{\infty}\underbrace{e^{-3t^2}\cos(6\pi t)\sin(2\pi f t)\,dt}_{\text{odd integrand results in zero integral}} \\
&= 2\int_{0}^{\infty} e^{-3t^2}\cos(6\pi t)\cos(2\pi f t)\,dt
\end{aligned}
\tag{10.83}
$$

Thus there is only the real part to plot. To make the graph we pick a value for *f*, say 3, calculate the integral $H(3)$, and plot the point. Then we will pick another value for *f*, say 4, calculate the integral $H(4)$, and plot the point. And so on. Although it appears that we have to pick values of *f* all the way from $-\infty$ to $+\infty$ (that's a lot of points!) two things save us. First notice that (10.83) shows, for example, that $H(3)$ and $H(-3)$ are identical, so we only have to pick positive values for *f*. Next notice that when $f = 3$ then (10.83) reads

$$H(3)=2\int_{0}^{\infty} e^{-3t^2}\cos^2(6\pi t)\,dt$$

but when $f \neq 3$, say when $f = 4$, then (10.83) reads

$$H(4) = 2\int_0^\infty e^{-3t^2}\cos(6\pi t)\cos(8\pi t)\,dt$$

The difference is that $\cos^2(6\pi t)$ is always positive so the integral for $H(3)$ is relatively large (it works out to equal 0.512) while $\cos(6\pi t)\cos(8\pi t)$ oscillates so the integral for $H(4)$ is relatively small (it equals 0.019). Thus we only have to compute $H(f)$ for values of f close to 3. On page 122 we listed the function called Simpson that does the integration. Paste that function into an Excel module (see the instructions on page 118 on how to do this). As well, paste the following code into the module.

```
Public Freq as Double                  'Freq is a global variable that all functions can 'see'
Public Const Pi = 3.14159265359        'Pi is a global constant that all functions can 'see'

Public Function Func(T)                'This is the integrand
   Func = 2 * Exp(-3 * T ^ 2) * Cos(6 * Pi * T) * Cos(2 * Pi * Freq * T)
End Function

Public Function H(F)                   'The spreadsheet calls this function
   Freq = F                            'Set a value for the frequency, Freq, used in the integrand
   H = Simpson(0, 10, 1000)            'Carry out the integral for the Fourier transform
End Function                           '10 is close enough to infinity.  Use 1000 strips.
```

Func is the integrand, which of course is different for each frequency. We achieve this by using a global variable for the frequency and using a "wrapper" function called $H(f)$ that sets the frequency in the integrand and then calls Simpson to do the integration. Because the integrand drops off so fast it is sufficient to just integrate from 0 to 10 rather than from 0 to ∞. However the integrand oscillates a lot so we have to use a lot of strips, say 1000. (If it wasn't for this you could use your calculator to do the integration.) With the help of these functions the spreadsheet is very simple. Column A contains some f values and column B calculates $H(f)$ for these values:

	A	B	C
1	***f***	***H*(*f*)**	**the formulas in column B**
2	0	1.41528E-13	=H(A2)
3	1	9.86048E-07	=H(A3)
4	2	0.019063943	=H(A4)
5	3	0.511663354	=H(A5)
6	4	0.019063943	=H(A6)
7	5	9.86048E-07	=H(A7)

Figure 10.37 The spreadsheet that calculates the Fourier transform $H(f)$.

Here is a plot of $H(f)$. Notice that it is a continuous curve sharply peaked at $f = 3$ and $f = -3$.

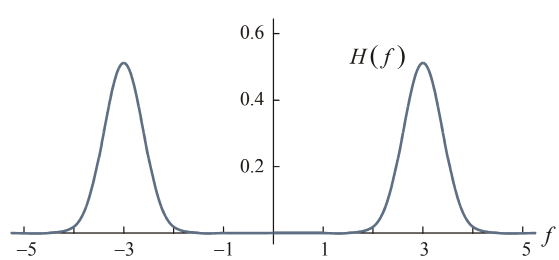


Figure 10.38 Plot of the real part of the Fourier transform, *H*(*f*). (The imaginary part is zero.)