Resonance

There are many systems that naturally vibrate or oscillate. Examples are a mass on the end of a spring, a pendulum, a child on a swing, a bouncing ball and an *RLC* circuit. For a system to have natural oscillations there are two requirements: (1) there must be a restoring force that pulls the system back toward equilibrium whenever it is displaced from equilibrium, and (2) the system must have inertia which causes it to overshoot equilibrium. In any oscillating system, to observe the natural oscillations all that is required is to displace the system from equilibrium and let it go. In each case there is a certain *natural angular velocity*, which we denote ω_n, for the oscillations.

Now suppose that an oscillating force is applied to the naturally oscillating system. If the angular velocity of the applied force, call it ω, is very different from the natural angular velocity ω_n, then the natural oscillations of the system will simply die out. But if the angular velocity ω of the applied force *equals* the natural angular velocity ω_n of the system then the oscillations will grow to a very large amplitude, limited only by the resistance or friction in the system. We say that the applied force and the system are then in **resonance**. When this happens there is a maximum transfer of energy from the source of the applied force to the system. ω_n is often also called ω_{res}, the angular velocity at which resonance occurs.

Fig. 8.18 shows an *RLC* circuit with the capacitor initially charged. When the switch is closed the capacitor discharges and a clockwise current flows as shown. But the capacitor doesn't simply discharge until its charge is zero. The inductor gives the current an '*inertia*' causing the system to overshoot this equilibrium and the capacitor becomes charged *in the opposite direction*. Repeated discharging and overshooting results in the natural oscillations in the current.

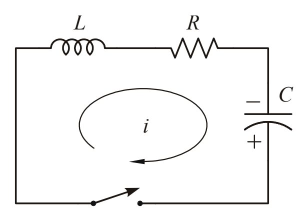

Figure 8.18 The current in a series *RLC* circuit can have natural oscillations.

The charged capacitor is analogous to a stretched spring and the inductor is analogous to the mass at the end of the spring. Our question is "what is ω_n for the *RLC* circuit and how can we find it?"

In Fig. 8.19 the same *R*, *L*, and *C* have been connected to an AC voltage source whose angular velocity ω can be varied at will. This voltage source acts as the oscillating applied force for this system.

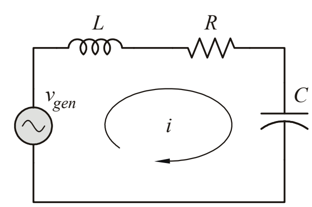

Figure 8.19 An alternating voltage source v_{gen} can excite the oscillations and drive them to a great amplitude.

If the angular velocity ω of the voltage source equals the natural angular velocity ω_n of the *LRC* circuit then the alternating current *i* will build up to the largest amplitude possible and there will be the maximum possible power transferred to the circuit. We will have resonance.

This is the peak in the graph in Fig. 8.20 where we have plotted the amplitude of *i* versus the angular velocity ω of the voltage generator.

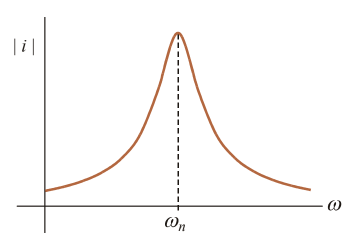

Figure 8.20 The amplitude of the current oscillations as a function of the angular velocity ω of v_{gen}. Adjusting ω to equal ω_n will cause large current oscillations. This condition is called resonance.

In our third book, *Differential Equations for Electrical Technology*, we will discover a more direct way of deriving ω_n but here we will derive it indirectly by finding what value of ω causes $|i|$ to be a maximum.

The amplitude of the current, $|i|$, is a maximum when the magnitude of the impedance, $|Z_{eq}|$, is a minimum, since $|i| = |v|/|Z_{eq}|$. The impedance when all three components, R, L and C are present is

$$Z_{eq} = Z_R + Z_L + Z_C$$

$$= R + j(X_L - X_C)$$

$$= R + j\left(\omega L - \frac{1}{\omega C}\right)$$

Fig. 8.21 shows Z_{eq} at a variety of values of ω. We see that $|Z_{eq}|$ is a minimum when ω is such that

$$X_L = X_C \quad \leftrightarrow \quad \omega L = \frac{1}{\omega C} \quad \leftrightarrow \quad \omega = \frac{1}{\sqrt{LC}}$$

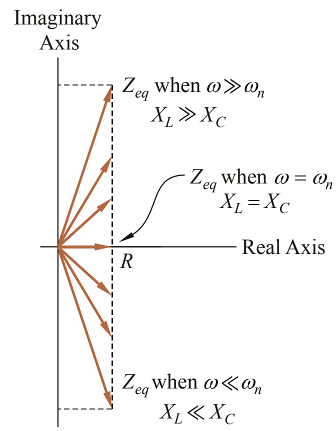

Figure 8.21 Z_{eq} for various values of ω. The shortest is when $X_L = X_C$.

This value of ω causes a maximum current and hence must be the natural or resonance angular velocity.

$$\omega_n = \frac{1}{\sqrt{LC}} \tag{8.15}$$

To learn about resonance check the app at phet.colorado.edu/en/simulation/resonance.