- If $F(z)$ has a **complex pole** at $z = b\angle\omega T$ (and therefore a complex conjugate pole at $z = b\angle-\omega T$ ) then $f(k)$ is an **exponential, oscillating sequence** of the form $b^k \sin(\omega T)$. The **magnitude** *b* of the pole determines the growth or decay of the sequence; and the **angle** $\omega T$ of the pole determines how quickly the sine factor completes a cycle. For example if $\omega T = 60°$ then when *k* = 6 the sin factor has completed $360°$ or one cycle, when *k* = 12 it has completed two cycles, and so on.

Here are six examples. The plots on the left show poles of $F(z)$ in the complex *z*-plane (marked with x's). The graphs on the right show the corresponding function $f(k)$.

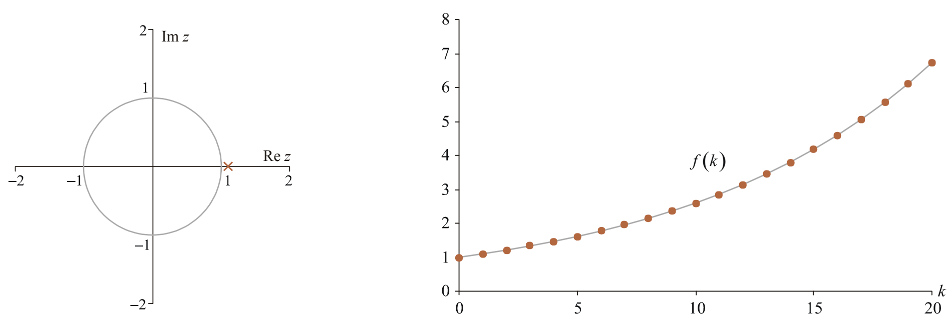


**Figure 5.3.** A real pole at *z* = 1.1. The result is an exponentially growing sequence $f(k)$.

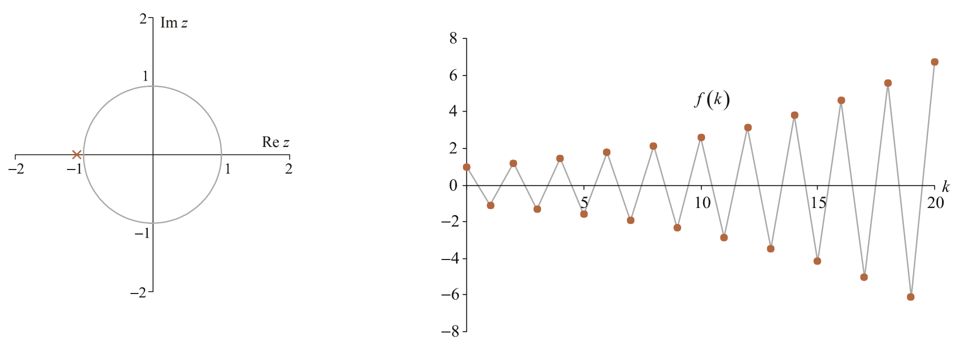


**Figure 5.4.** A pole at *z* = –1.1. The result is an alternating, exponentially growing sequence.

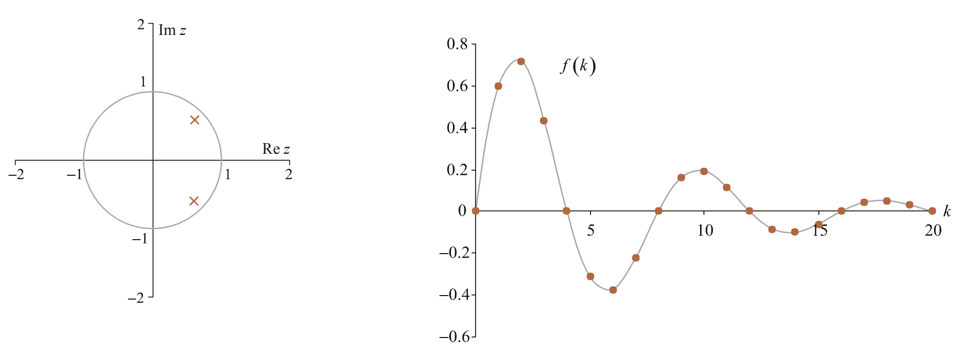


**Figure 5.5** Complex pole at $z = 0.85\angle 45°$. Result is an exponentially decaying oscillation. The sin factor completes one cycle when *k* = 8.

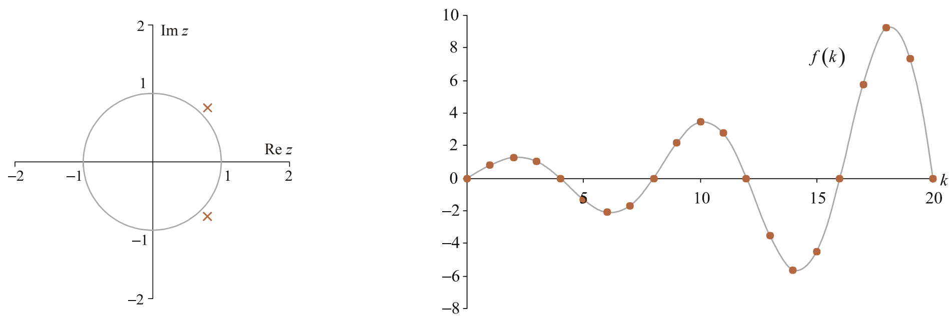


**Figure 5.6** Complex pole at $z = 1.15\angle 45°$. Result is an exponentially growing oscillation.

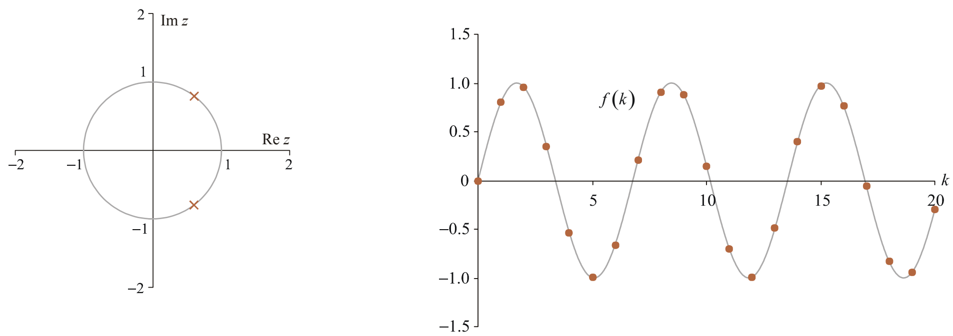


**Figure 5.7** Complex pole at $z = 1\angle 53°$. The result is an oscillation that cycles when $k \approx 7$.

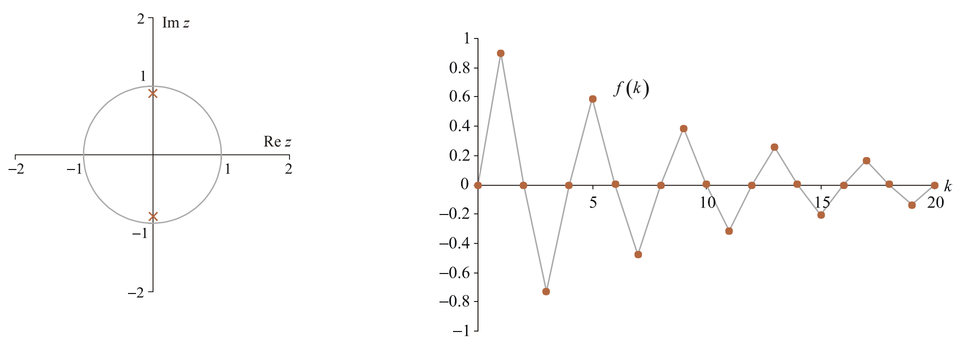


**Figure 5.8** Imaginary pole at $z = 0.9\angle 90°$. The result is an exponentially decaying oscillation that repeats when *k* = 4 (and is therefore zero for all even values of *k*).

**The connection between Laplace transforms and *z*-transforms.**

Imagine that $f(t)$ is a continuous function of time but that we only sample it at time intervals of size *T*. If $f(kT)$ denotes the value of the *k*-th sample, then the *z*-transform of this function is defined as

$$Z\left(f(kT)\right) = \sum_{k=0}^{\infty} f(kT) z^{-k} . \tag{5.13}$$