

# Table of Contents

<b>1. Vectors and Curves in Space .....</b>	<b>5</b>
1.1 Vectors .....	7
1.2 Dot Product .....	12
1.3 Cross Product .....	17
1.4 Curves in Space .....	21
1.5 Motion in Space .....	26
1.6 Arc Length .....	29
Answers, Chapter 1 – Vectors and Curves in Space .....	33
<b>2. Partial Derivatives .....</b>	<b>35</b>
2.1 Functions of Several Variables .....	35
2.2 Partial Derivatives .....	37
2.3 Chain Rule for Partial Derivatives .....	37
2.4 Directional Derivatives and Gradients .....	41
2.5 Differentials and Tangent Planes to Surfaces .....	43
2.6 Higher Order Partial Derivatives .....	49
2.7 Taylor Series for Functions of Several Variables .....	50
2.8 Local Maxima and Minima of a Function of Two Variables .....	53
2.9 Method of Lagrange Multipliers .....	58
Answers, Chapter 2 – Partial Derivatives .....	63
<b>3. Multiple Integrals .....</b>	<b>67</b>
3.1 Review of Definite Integrals .....	67
3.2 Double Integrals .....	68
3.3 Using Iterated Integrals to Evaluate Double Integrals .....	69
3.4 Triple Integrals .....	76
3.5 Using Iterated Integrals to Evaluate Triple Integrals .....	76
3.6 Integration Using Polar, Cylindrical and Spherical Coordinates .....	80
Answers, Chapter 3 – Multiple Integrals .....	85
<b>4. Vector Fields, Grad, Div and Curl .....</b>	<b>87</b>
4.1 Understanding the Vector Field Notation .....	88
4.2 Particle Paths .....	90
4.3 Divergence of a Vector Field .....	92
4.4 Divergence in Fluid Flow .....	92
4.5 Curl of a Vector Field .....	94
4.6 Some Properties of Grad, Div and Curl .....	96
4.7 Grad, Div and Curl in Cylindrical and Spherical Coordinates .....	97
4.8 Vector Fields that Possess a Potential .....	101
4.9 The Electric Field and the Voltage Field .....	103
Answers, Chapter 4 – Vector Fields, Grad, Div and Curl .....	108
<b>5. Line Integrals .....</b>	<b>109</b>
5.1 The Line Integral .....	109
5.2 Line Integrals that are Independent of Path .....	113
Answers, Chapter 5 – Line Integrals .....	117

<b>6. Green's Theorem .....</b>	<b>119</b>
Answers, Chapter 6 – Green's Theorem .....	127
<b>7. Surface Integrals .....</b>	<b>129</b>
7.1 Surface Area.....	129
7.2 Surface Integrals .....	136
Answers, Chapter 7 – Surface Integrals.....	142
<b>8. Gauss' and Stokes' Theorems.....</b>	<b>143</b>
8.1 Flow and an Alternative Definition of Divergence .....	143
8.2 Gauss' Theorem .....	144
8.3 Examples using Gauss' Theorem.....	147
8.4 Circulation and an Alternative Definition of Curl.....	151
8.5 Stokes' Theorem .....	152
8.6 Examples using Stokes' Theorem.....	155
Answers, Chapter 8 – Gauss' and Stokes' Theorems .....	162
<b>9. Partial Differential Equations.....</b>	<b>163</b>
9.1 Introduction .....	163
9.2 Some Important Partial Differential Equations .....	165
9.3 Solving PDEs using Separation of Variables .....	184
9.4 Solving Laplace's and Poisson's equations on a Grid.....	189
9.5 Solving the Wave Equation using the Finite Difference Method.....	197
9.6 Solving the Heat Equation using the Crank-Nicholson Method .....	205
9.7 Simulating EM Waves using the Finite Difference Method .....	213
Answers, Chapter 9 – Partial Differential Equations .....	223